

Edexcel Maths C3

Past Paper Pack

2005-2013

Leave
blank

2. (a) Differentiate with respect to x

(i) $3 \sin^2 x + \sec 2x,$ (3)

(ii) $\{x + \ln(2x)\}^3.$ (3)

Given that $y = \frac{5x^2 - 10x + 9}{(x - 1)^2}, \quad x \neq 1,$

(b) show that $\frac{dy}{dx} = -\frac{8}{(x-1)^3}.$ (6)



3. The function f is defined by

$$f:x \rightarrow \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \frac{2}{x-1}$, $x > 1$. (4)

(b) Find $f^{-1}(x)$. (3)

The function g is defined by

$$g:x \rightarrow x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$. (3)



5. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A. \tag{2}$$

(b) Show that

$$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3). \tag{4}$$

(c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (4)

(d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate. (5)



6.

Figure 1

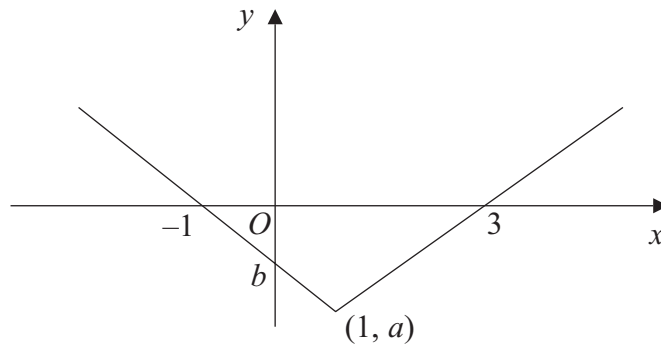


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)



Leave
blank

Question 6 continued



7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1 + ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

- (a) show that $a = 0.12$, (3)

- (b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850. (4)

- (c) Show that $p = \frac{336}{0.12 + e^{-0.2t}}$. (1)

- (d) Hence show that the population cannot exceed 2800. (2)



Centre No.						Paper Reference					Surname	Initial(s)		
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

**Edexcel GCE
Core Mathematics C3
Advanced Level**

Monday 23 January 2006 – Afternoon
Time: 1 hour 30 minutes

Examiner’s use only

--	--	--

Team Leader’s use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination Items included with question papers
 Mathematical Formulae (Green) Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
 Check that you have the correct question paper.
 You must write your answer for each question in the space following the question.
 When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
 Full marks may be obtained for answers to ALL questions.
 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 8 questions in this question paper. The total mark for this paper is 75.
 There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
 You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy. ©2006 Edexcel Limited.

Printer’s Log. No.
N23495A



Turn over

1.

Figure 1

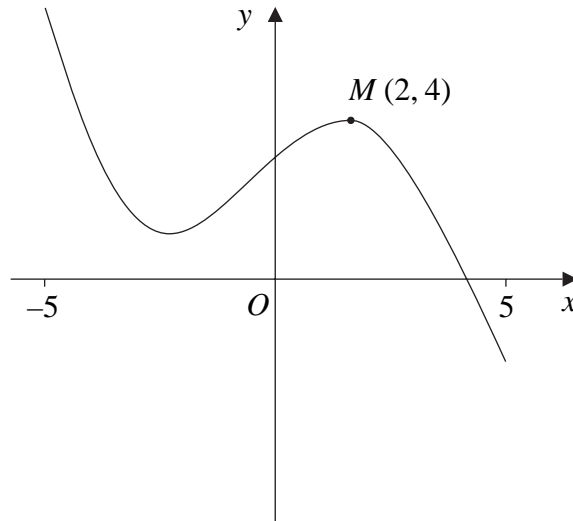


Figure 1 shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.
The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x) + 3$, (2)

(b) $y = |f(x)|$, (2)

(c) $y = f(|x|)$. (3)

Show on each graph the coordinates of any maximum turning points.



Leave
blank

Question 1 continued

Q1

(Total 7 marks)



3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x -coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)



Leave blank

4. (a) Differentiate with respect to x

(i) x^2e^{3x+2} , (4)

(ii) $\frac{\cos(2x^3)}{3x}$. (4)

(b) Given that $x = 4 \sin(2y + 6)$, find $\frac{dy}{dx}$ in terms of x . (5)



5.

$$f(x) = 2x^3 - x - 4.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}. \tag{3}$$

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3 . (3)

The only real root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places. (3)



6. $f(x) = 12 \cos x - 4 \sin x$.

Given that $f(x) = R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90^\circ$,

(a) find the value of R and the value of α . (4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leq x < 360^\circ$, giving your answers to one decimal place. (5)

(c) (i) Write down the minimum value of $12 \cos x - 4 \sin x$. (1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs. (2)



7. (a) Show that

$$(i) \frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \frac{1}{4})\pi, n \in \mathbb{Z}, \tag{2}$$

$$(ii) \frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}. \tag{3}$$

(b) Hence, or otherwise, show that the equation

$$\cos \theta \left(\frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta. \tag{3}$$

(c) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π . (4)



8. The functions f and g are defined by

$$f: x \rightarrow 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g: x \rightarrow e^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function gf is

$$gf: x \rightarrow 4e^{4x}, \quad x \in \mathbb{R}. \tag{4}$$

(b) In the space provided on page 19, sketch the curve with equation $y = gf(x)$, and show the coordinates of the point where the curve cuts the y -axis. (1)

(c) Write down the range of gf . (1)

(d) Find the value of x for which $\frac{d}{dx}[gf(x)] = 3$, giving your answer to 3 significant figures. (4)



3.

Figure 1

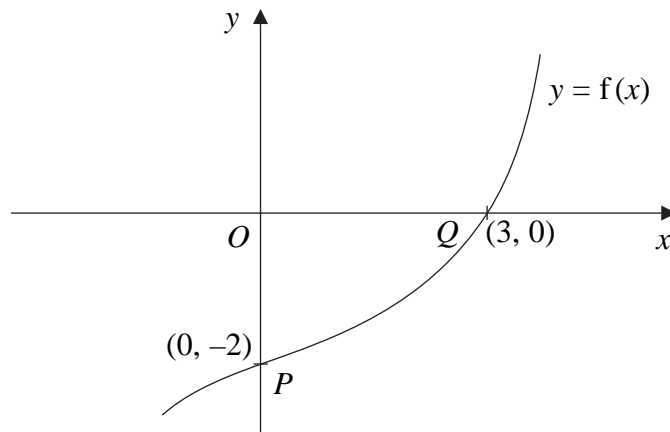


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2} f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



Leave
blank

Question 3 continued



5.

Figure 2

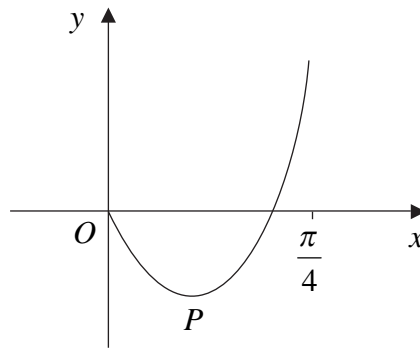


Figure 2 shows part of the curve with equation

$$y = (2x - 1) \tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P . The x -coordinate of P is k .

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k .

(b) Calculate the values of x_1 , x_2 , x_3 and x_4 , giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)



7. For the constant k , where $k > 1$, the functions f and g are defined by

$$\begin{aligned} f: x &\mapsto \ln(x+k), & x > -k, \\ g: x &\mapsto |2x-k|, & x \in \mathbb{R}. \end{aligned}$$

(a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f .

(1)

(c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k .

(4)



Leave
blank

Question 8 continued

Q8

--	--

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END



Leave
blank

Question 4 continued

Lined area for writing the answer to Question 4.



Leave
blank

5.

Figure 1

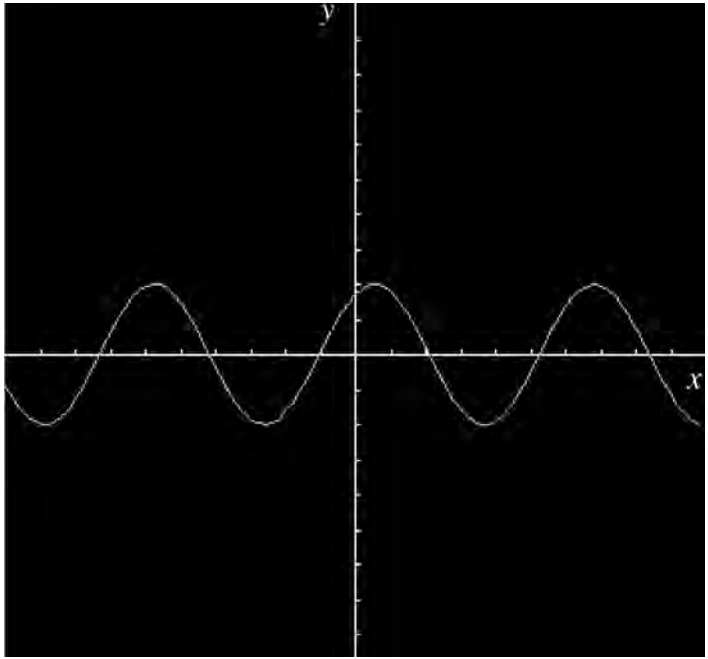


Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3} \cos x + \sin x.$$

- (a) Express the equation of the curve in the form $y = R \sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

- (b) Find the values of x , $0 \leq x < 2\pi$, for which $y = 1$. (4)



6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^x$$

and write down the domain of f^{-1} .

(4)

(b) Write down the range of f^{-1} .

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k .

(d) Calculate the values of x_1 and x_2 , giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)



Leave
blank

Question 6 continued



Leave
blank

Question 7 continued

Q7

(Total 13 marks)



Centre No.					Paper Reference							Surname	Initial(s)
Candidate No.					6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 14 June 2007 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature. Check that you have the correct question paper. You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.



Turn over

Leave blank

4. $f(x) = -x^3 + 3x^2 - 1.$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}. \tag{2}$$

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2 , x_3 and x_4 , giving all your answers to 4 decimal places. (2)

(c) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places. (3)



5. The functions f and g are defined by

$$f : x \mapsto \ln(2x-1), \quad x \in \mathbb{R}, x > \frac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, x \neq 3.$$

- (a) Find the exact value of $fg(4)$. (2)

- (b) Find the inverse function $f^{-1}(x)$, stating its domain. (4)

- (c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y -axis. (3)

- (d) Find the exact values of x for which $\left| \frac{2}{x-3} \right| = 3$. (3)



Leave blank

6. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. (2)

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places. (5)



7. (a) Prove that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n^\circ.$$

(4)

(b) On the axes on page 20, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$.

(2)

(c) Solve, for $0^\circ < \theta < 360^\circ$, the equation

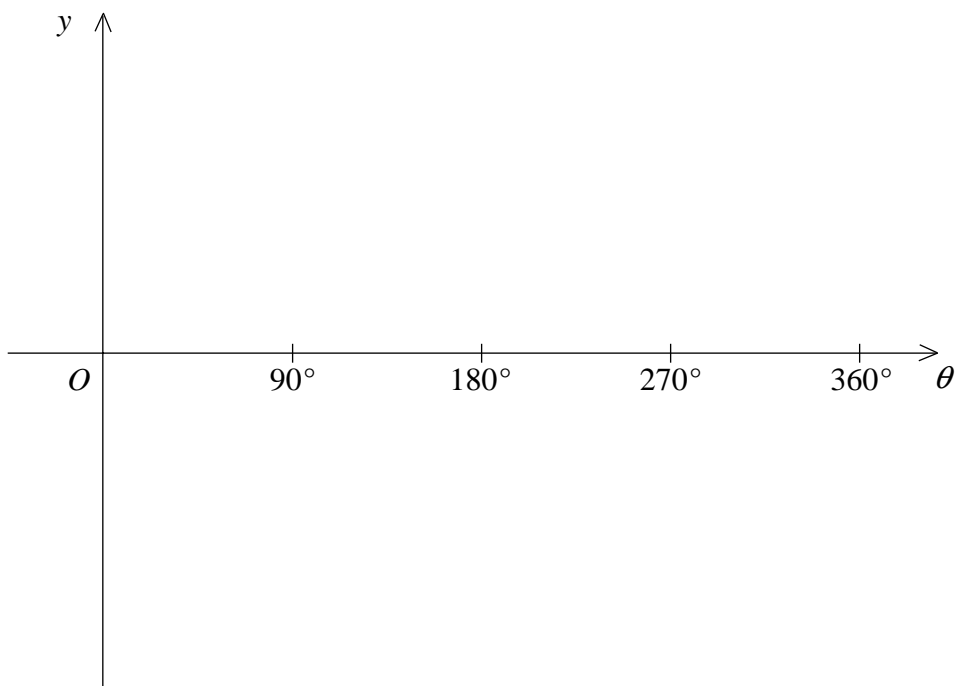
$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 3,$$

giving your answers to 1 decimal place.

(6)



Question 7 continued





- 8. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

- (a) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

- (b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

- (c) Find the value of T .

(3)



Centre No.						Paper Reference					Surname	Initial(s)	
Candidate No.					6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 17 January 2008 – Afternoon
Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. Write your answers in the spaces provided in this question paper. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.



3. $f(x) = \ln(x + 2) - x + 1, \quad x > -2, x \in \mathbb{R}.$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$. (2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places. (3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places. (2)

Lined area for student answers.



4.

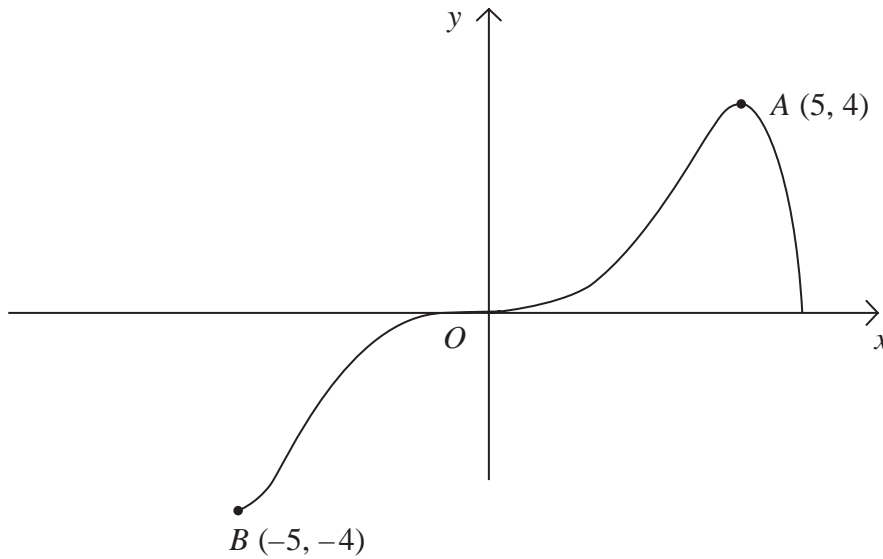


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$, (3)

(b) $y = f(|x|)$, (3)

(c) $y = 2f(x+1)$. (4)

On each sketch, show the coordinates of the points corresponding to A and B .



Leave
blank

Question 4 continued



5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay. (1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures. (4)

(c) Calculate the number of atoms that will be left when $t = 22\,920$. (2)

(d) In the space provided on page 13, sketch the graph of R against t . (2)



6. (a) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

- (b) (i) Prove that

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \equiv 2 \sec x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

- (ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 4.$$

(3)



Question 6 continued

Handwritten answer area containing several lines of calculations and diagrams. The text is mostly illegible due to blurring.



7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \quad -\pi \leq x \leq \pi.$$

The point $A(0, 4)$ lies on C .

(a) Find an equation of the normal to the curve C at A . (5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give the value of α to 3 significant figures. (4)

(c) Find the coordinates of the points of intersection of the curve C with the x -axis.
Give your answers to 2 decimal places. (4)



8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1} . (2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$. (2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$. (5)



Leave
blank

Question 8 continued

Handwritten area for the answer to Question 8, containing numerous blank horizontal lines for writing.

(Total 13 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END



1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

- (a) Find, in terms of $\ln 2$, the x -coordinate of P . (2)

- (b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found. (4)



3.

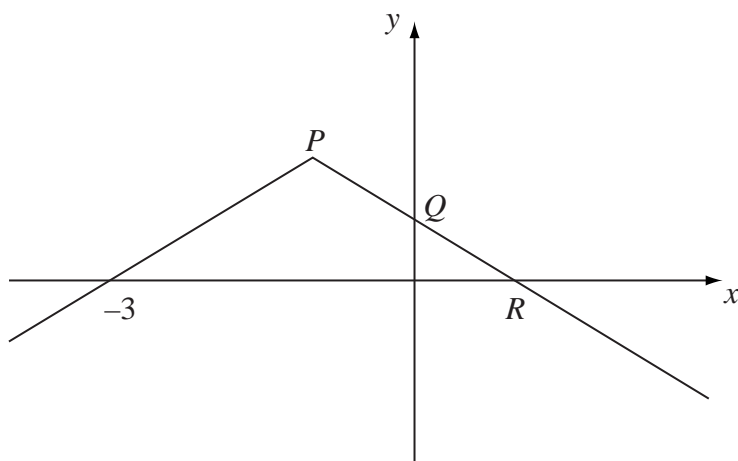


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point P .

The graph cuts the y -axis at the point Q and the x -axis at the points $(-3, 0)$ and R .

Sketch, on separate diagrams, the graphs of

(a) $y = |f(x)|$, (2)

(b) $y = f(-x)$. (2)

Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P , Q and R , (3)

(d) solve $f(x) = \frac{1}{2}x$. (5)



4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}$, $x > 3$. (4)

(b) Find the range of f . (2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function. (3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$. (3)



6. (a) Differentiate with respect to x ,

(i) $e^{3x}(\sin x + 2 \cos x)$, (3)

(ii) $x^3 \ln(5x + 2)$. (3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)



Leave
blank

Question 6 continued

Handwriting practice area consisting of 35 horizontal lines.



7.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0=1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)



Leave
blank

Question 7 continued

Blank lined area for writing the answer to Question 7.

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END



Centre No.						Paper Reference	Surname	Initial(s)
							Signature	
Candidate No.						6 6 6 5 / 0 1		

Paper Reference(s)
6665/01

Edexcel GCE
Core Mathematics C3
Advanced

Thursday 15 January 2009 – Morning
Time: 1 hour 30 minutes

Examiner's use only		
Team Leader's use only		

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination **Items included with question papers**
 Mathematical Formulae (Green) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
 Answer ALL the questions.
 You must write your answer for each question in the space following the question.
 When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
 Full marks may be obtained for answers to ALL questions.
 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 8 questions in this question paper. The total mark for this paper is 75.
 There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
 You should show sufficient working to make your methods clear to the Examiner.
 Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy. ©2009 Edexcel Limited.



3.

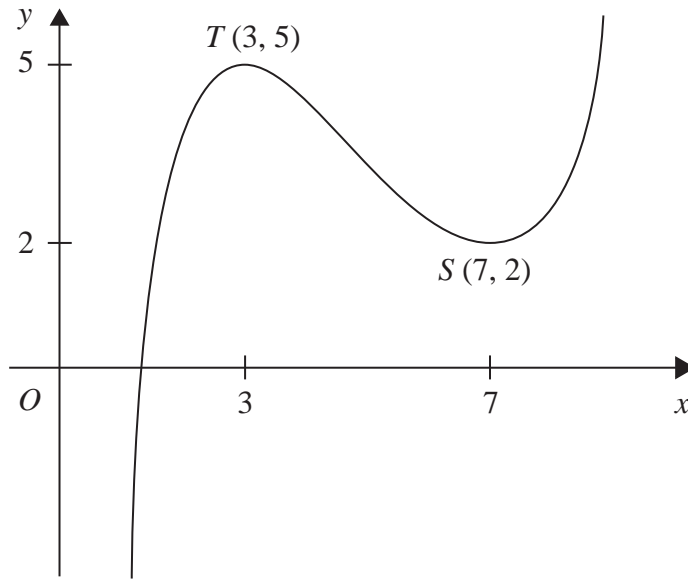


Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.
 The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$, (3)

(b) $y = |f(x)|$. (3)

Indicate on each diagram the coordinates of any turning points on your sketch.



5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{-x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g . (1)

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{-x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg . (1)

(d) Solve the equation $\frac{d}{dx}[fg(x)] = x(xe^{x^2} + 2)$. (6)



Leave blank

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \tag{4}$$

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π . (5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}). \tag{4}$$

Lined area for student answers.



7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

- (a) Find the exact coordinates of P . (5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

- (b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3 . (3)

- (c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places. (3)



8. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)^\circ + 4 \sin(15t)^\circ,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)



Centre No.						Paper Reference						Surname	Initial(s)	
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

**Edexcel GCE
Core Mathematics C3
Advanced**

Thursday 11 June 2009 – Morning
Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Materials required for examination
Mathematical Formulae (Orange or Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy. ©2009 Edexcel Limited.



Turn over

1.

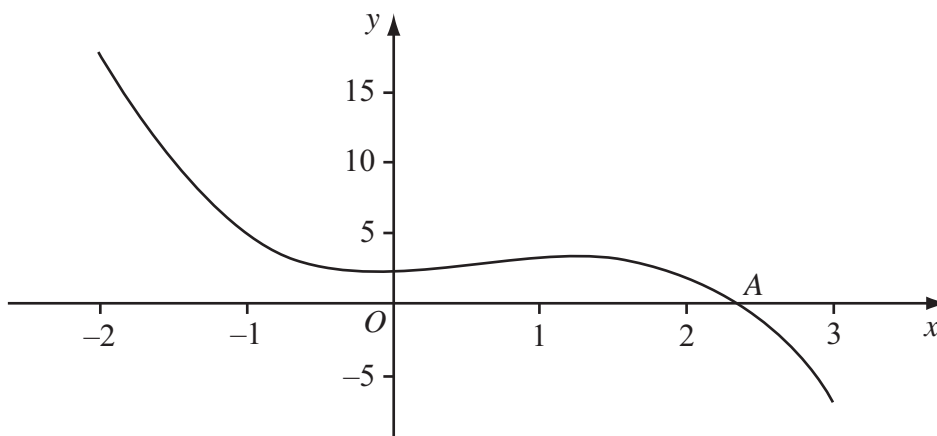


Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x -axis at the point A where $x = \alpha$.

To find an approximation to α , the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4 .
Give your answers to 3 decimal places where appropriate. (3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places. (3)



2. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$. (2)

(b) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$2 \tan^2\theta + 4 \sec\theta + \sec^2\theta = 2 \quad (6)$$



3. Rabbits were introduced onto an island. The number of rabbits, P , t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geq 0$$

(a) Write down the number of rabbits that were introduced to the island. (1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000. (2)

(c) Find $\frac{dP}{dt}$. (2)

(d) Find P when $\frac{dP}{dt} = 50$. (3)



5.

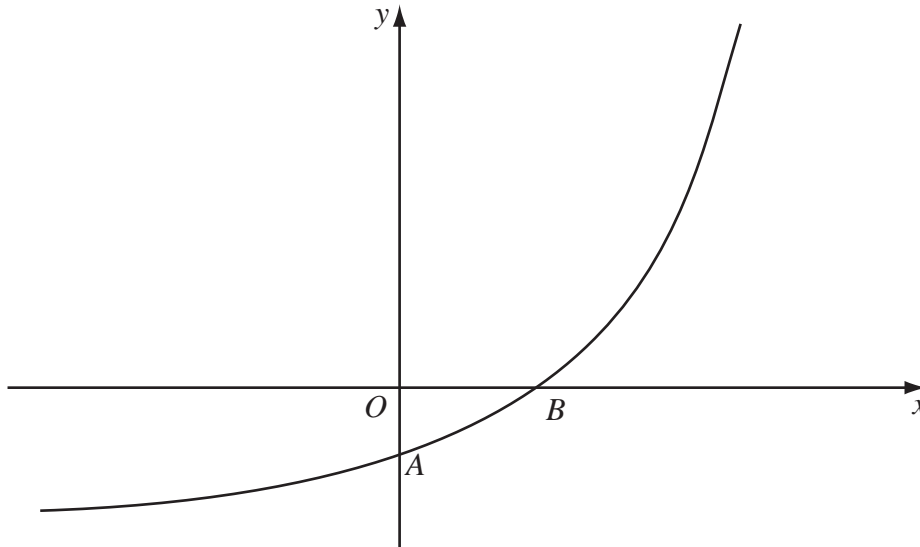


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2} \ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$. (2)

Show on each sketch the coordinates, in terms of k , of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f , (1)

(d) find $f^{-1}(x)$, (3)

(e) write down the domain of f^{-1} . (1)



Leave
blank

Question 5 continued



6. (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2 \sin^2 A \tag{2}$$

The curves C_1 and C_2 have equations

$$C_1: y = 3 \sin 2x$$

$$C_2: y = 4 \sin^2 x - 2 \cos 2x$$

- (b) Show that the x -coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4 \cos 2x + 3 \sin 2x = 2 \tag{3}$$

- (c) Express $4 \cos 2x + 3 \sin 2x$ in the form $R \cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

- (d) Hence find, for $0 \leq x < 180^\circ$, all the solutions of

$$4 \cos 2x + 3 \sin 2x = 2$$

giving your answers to 1 decimal place.

(4)



8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$. (1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places. (5)



2.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

(a) Show that $f(x) = 0$ can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

(2)

The equation $f(x) = 0$ has one positive root α .

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α .

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2 , x_3 and x_4 .

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)



5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)



6.

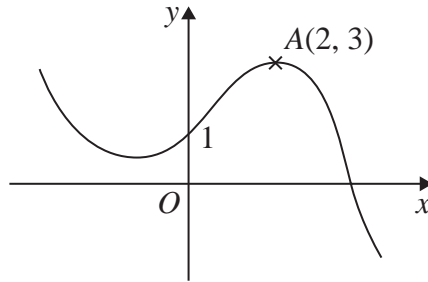
**Figure 1**

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y -axis at the point $(0, 1)$ and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

- (i) $y = f(-x) + 1$,
- (ii) $y = f(x + 2) + 3$,
- (iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y -axis and the coordinates of the point to which A is transformed.

(9)

Leave
blank

Question 6 continued



7. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$. (3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{dy}{dx}$. (4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b) .

(c) Find the values of the constants a and b , giving your answers to 3 significant figures. (4)



Leave
blank

Question 7 continued

Lined area for writing the answer to Question 7.



9. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$ **(3)**

(b) $3^x e^{7x+2} = 15$ **(5)**

(ii) The functions f and g are defined by

$$\begin{aligned} f(x) &= e^{2x} + 3, & x &\in \mathbb{R} \\ g(x) &= \ln(x - 1), & x &\in \mathbb{R}, x > 1 \end{aligned}$$

(a) Find f^{-1} and state its domain. **(4)**

(b) Find fg and state its range. **(3)**



Leave
blank

Question 9 continued

[This section contains 25 horizontal lines for writing the answer to Question 9.]

Q9

--	--

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END



1. (a) Show that

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta \tag{2}$$

(b) Hence find, for $-180^\circ \leq \theta < 180^\circ$, all the solutions of

$$\frac{2 \sin 2\theta}{1 + \cos 2\theta} = 1$$

Give your answers to 1 decimal place. (3)



4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes. (2)

(b) Solve $f(x) = 15 + x$. (3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

(c) Find $fg(2)$. (2)

(d) Find the range of g . (3)



5.

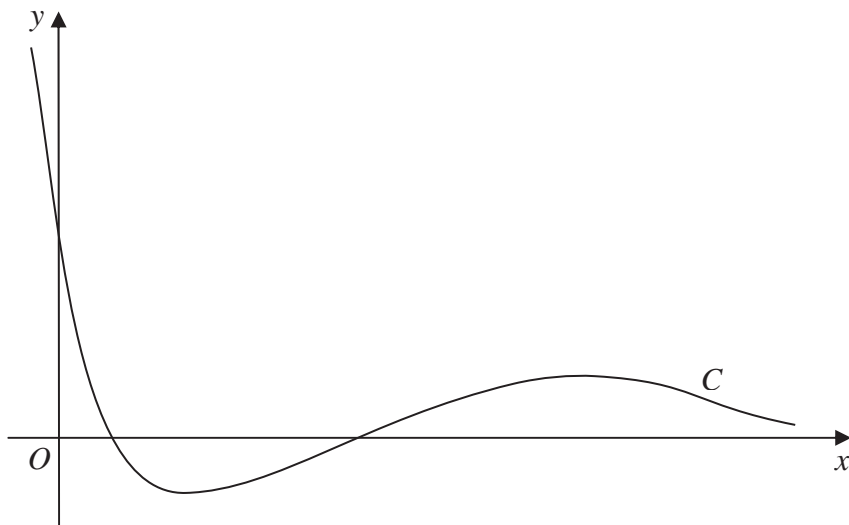


Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

- (a) Find the coordinates of the point where C crosses the y -axis. (1)
- (b) Show that C crosses the x -axis at $x = 2$ and find the x -coordinate of the other point where C crosses the x -axis. (3)
- (c) Find $\frac{dy}{dx}$. (3)
- (d) Hence find the exact coordinates of the turning points of C . (5)



6.

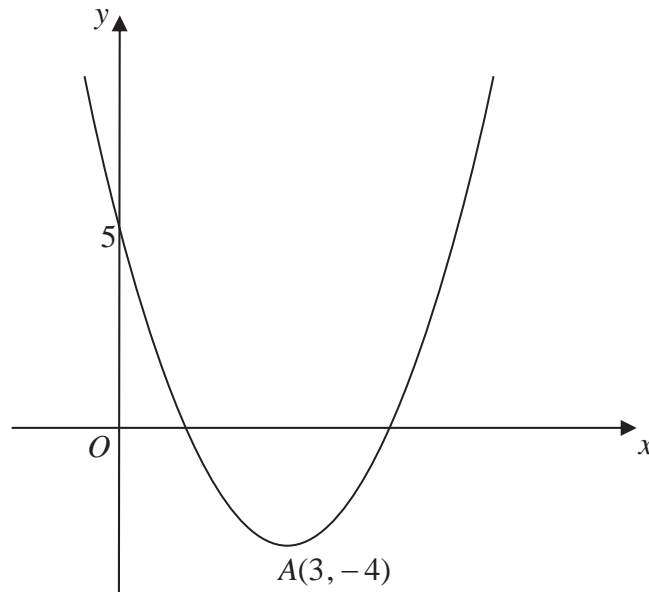


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation

$$y = f(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y -axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)



Leave blank

7. (a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)



Leave
blank

Question 7 continued

A series of horizontal lines for writing the answer to Question 7.



Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.					6	6	6	5	/	0	1	Signature		

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 24 January 2011 – Morning
Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

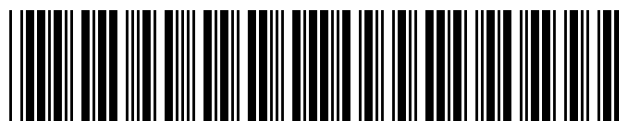
Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy. ©2011 Edexcel Limited.



Turn over

1. (a) Express $7 \cos x - 24 \sin x$ in the form $R \cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
 Give the value of α to 3 decimal places.

(3)

- (b) Hence write down the minimum value of $7 \cos x - 24 \sin x$.

(1)

- (c) Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10$$

giving your answers to 2 decimal places.

(5)



2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $f(x)$ and find $f'(2)$.

(3)



4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, $\theta^\circ\text{C}$, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C ,

- (a) find the value of A . (2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C .

- (b) Show that $k = \frac{1}{5} \ln 2$. (3)

- (c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in $^\circ\text{C}$ per minute, to 3 decimal places. (3)



5.

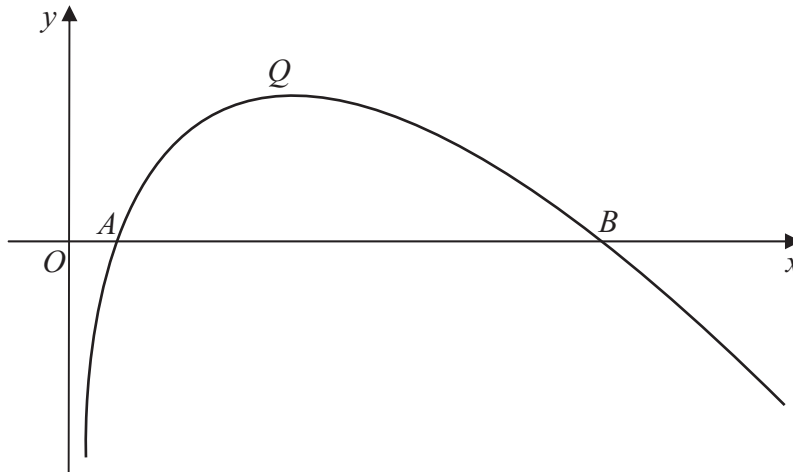


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of
$$x = \frac{8}{1 + \ln x}$$
 (3)

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)



6. The function f is defined by

$$f: x \mapsto \frac{3 - 2x}{x - 5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

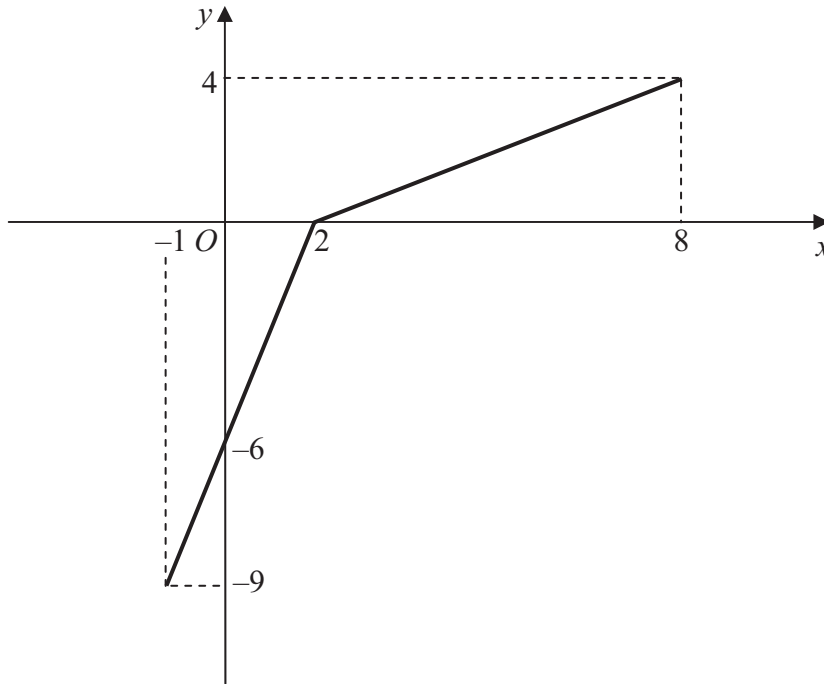


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|,$

(ii) $y = g^{-1}(x).$

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)



Leave
blank

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)



Leave blank

8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y .

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x .

(4)



2. $f(x) = 2 \sin(x^2) + x - 2, \quad 0 \leq x < 2\pi$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$ (2)

The equation $f(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x) \right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n) \right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1 , x_2 and x_3 , giving your answers to 5 decimal places. (3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places. (3)



3.

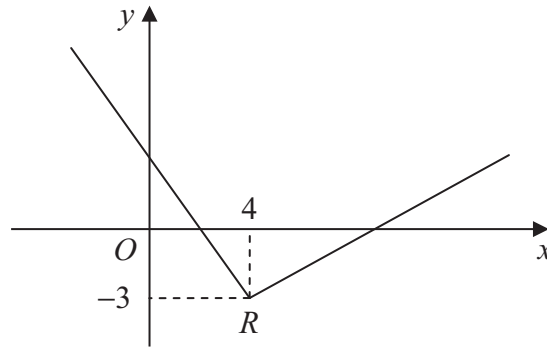


Figure 1

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x+4)$, **(3)**

(b) $y = |f(-x)|$. **(3)**

On each diagram, show the coordinates of the point corresponding to R .



Leave
blank

4. The function f is defined by

$$f : x \mapsto 4 - \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1$$

(a) Find $f^{-1}(x)$. **(3)**

(b) Find the domain of f^{-1} . **(1)**

The function g is defined by

$$g : x \mapsto e^{-x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form. **(3)**

(d) Find the range of fg . **(1)**



Leave
blank

6. (a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \quad \theta \neq 90n^\circ, \quad n \in \mathbb{Z} \tag{4}$$

(b) Hence, or otherwise,

(i) show that $\tan 15^\circ = 2 - \sqrt{3}$, **(3)**

(ii) solve, for $0 < x < 360^\circ$,

$$\operatorname{cosec} 4x - \cot 4x = 1 \tag{5}$$



Leave blank

8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R \cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. **(4)**

$$f(x) = e^{2x} \cos 3x$$

- (b) Show that $f'(x)$ can be written in the form

$$f'(x) = R e^{2x} \cos(3x + \alpha)$$

where R and α are the constants found in part (a). **(5)**

- (c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point. **(3)**



Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 23 January 2012 – Morning
Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination **Items included with question papers**
 Mathematical Formulae (Pink) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Pearson Education Ltd copyright policy. ©2012 Pearson Education Ltd.

Printer's Log No. **P40084A**

W850/R6665/57570 5/7/5/



Turn over



2.

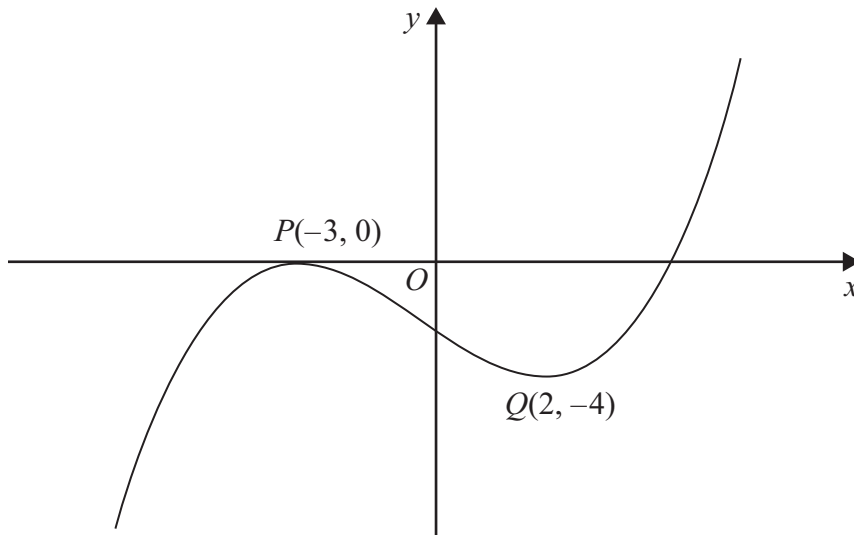


Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$ **(3)**

(b) $y = |f(x)|$ **(3)**

On each diagram, show the coordinates of any stationary points.



Leave blank

5. Solve, for $0 \leq \theta \leq 180^\circ$,

$$2\cot^2 3\theta = 7 \operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)



6. $f(x) = x^2 - 3x + 2 \cos(\frac{1}{2}x), \quad 0 \leq x \leq \pi$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$ **(2)**

The curve with equation $y = f(x)$ has a minimum point P .

(b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin(\frac{1}{2}x)}{2} \tag{4}$$

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin(\frac{1}{2}x_n)}{2}, \quad x_0 = 2$$

find the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. **(3)**

(d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places. **(3)**



Leave blank

7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ (4)

(b) Find $f^{-1}(x)$ (3)

(c) Find the domain of f^{-1} (1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e . (4)



8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \tag{4}$$

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta} \tag{3}$$

(c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π . (6)



1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)



2. $f(x) = x^3 + 3x^2 + 4x - 12$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3 \qquad (3)$$

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1 , x_2 and x_3 . (3)

The root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places. (3)



3.

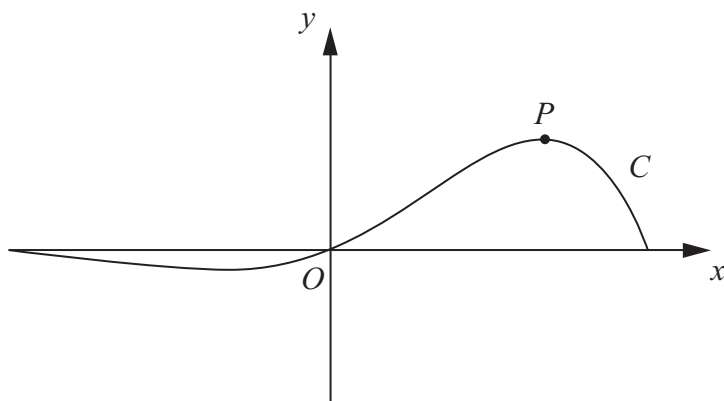


Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$$

- (a) Find the x coordinate of the turning point P on C , for which $x > 0$.
Give your answer as a multiple of π .

(6)

- (b) Find an equation of the normal to C at the point where $x = 0$

(3)



Leave
blank

Question 3 continued

Handwritten notes and calculations for Question 3 continued, including a derivation of the relationship between pressure, force, and area, and a calculation of the cross-sectional area of a wire.

The first part of the notes shows the derivation of the equation $p = \frac{F}{A}$ from the definition of pressure $p = \frac{F}{A}$ and the relationship $F = \frac{W}{h}$. It then combines these to get $p = \frac{W}{Ah}$.

The second part of the notes shows a calculation for the cross-sectional area A of a wire. It starts with $p = \frac{F}{A}$ and rearranges to $A = \frac{F}{p}$. It then substitutes $F = 10N$ and $p = 1.5 \times 10^8 \text{ Pa}$ to get $A = \frac{10}{1.5 \times 10^8}$. The final result is $A = 6.67 \times 10^{-8} \text{ m}^2$.



4.

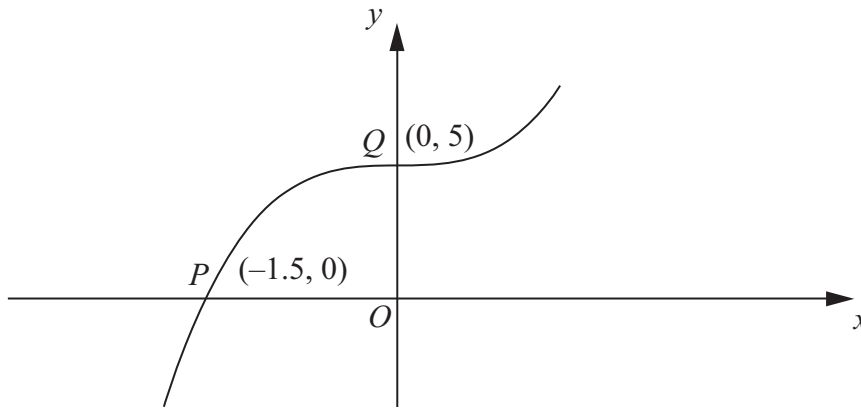


Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$
 The curve passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$ **(2)**

(b) $y = f(|x|)$ **(2)**

(c) $y = 2f(3x)$ **(3)**

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



Leave
blank

Question 4 continued



6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \quad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \quad x > 0$$

- (a) State the range of f . (1)
- (b) Find $fg(x)$, giving your answer in its simplest form. (2)
- (c) Find the exact value of x for which $f(2x+3) = 6$ (4)
- (d) Find f^{-1} , the inverse function of f , stating its domain. (3)
- (e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. (4)



8. $f(x) = 7 \cos 2x - 24 \sin 2x$

Given that $f(x) = R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the value of R and the value of α . (3)

(b) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180^\circ$, giving your answers to 1 decimal place. (5)

(c) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a , b , and c are constants to be found. (2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x$$
(2)



Leave blank

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w , **(2)**

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants. **(5)**



2.

$$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \quad x < 6 \tag{2}$$

The root of $g(x) = 0$ is α .

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2$$

is used to find an approximate value for α .

(b) Calculate the values of x_1 , x_2 and x_3 to 4 decimal places. (3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places. (3)



3.

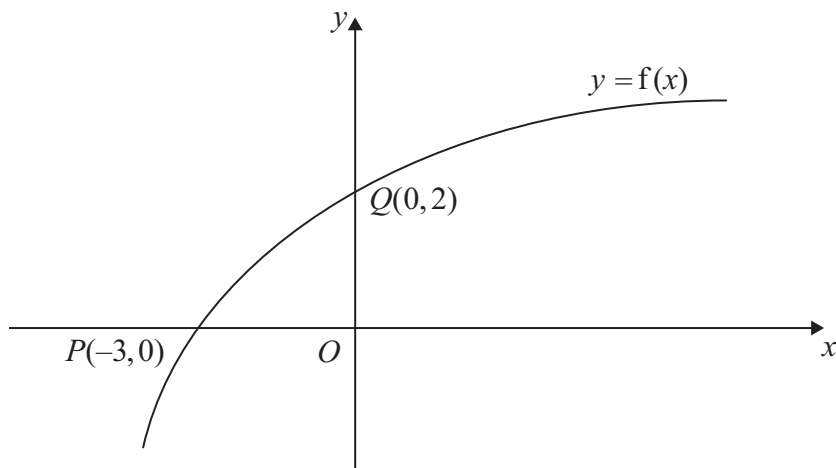


Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $ff(-3)$. (2)

On separate diagrams, sketch the curve with equation

(b) $y = f^{-1}(x)$, (2)

(c) $y = f(|x|) - 2$, (2)

(d) $y = 2f\left(\frac{1}{2}x\right)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



Leave
blank

Question 3 continued



Leave
blank

4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b)
$$p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi$$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)



Leave blank

6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5^\circ + \cos 22.5^\circ)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

(b) Hence solve, for $0 \leq \theta < 360^\circ$, the equation

$$\cos 2\theta + \sin \theta = 1$$

(4)

Lined area for student answers.



Leave blank

7.
$$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0$$

(a) Show that $h(x) = \frac{2x}{x^2+5}$ (4)

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. (3)

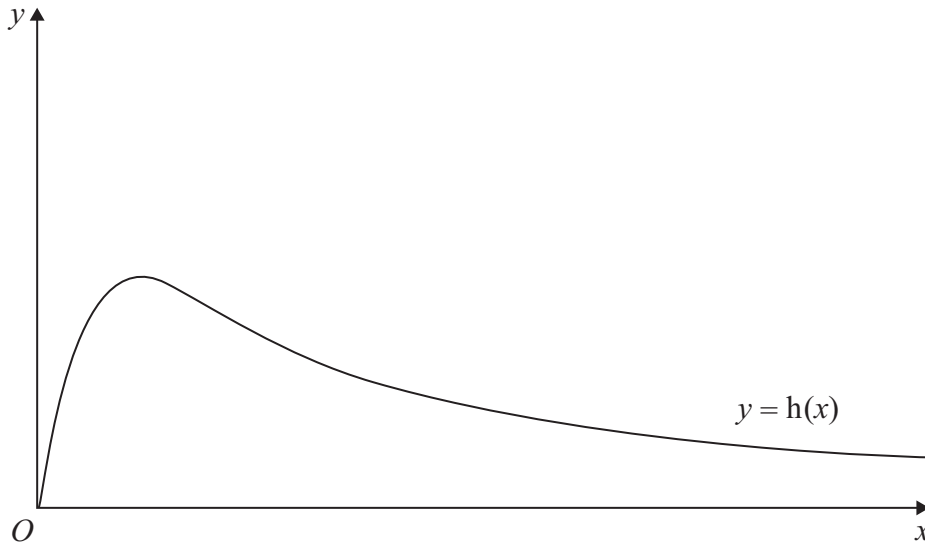


Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. (5)



Question 7 continued

Lined area for writing the answer to Question 7 continued.



2.

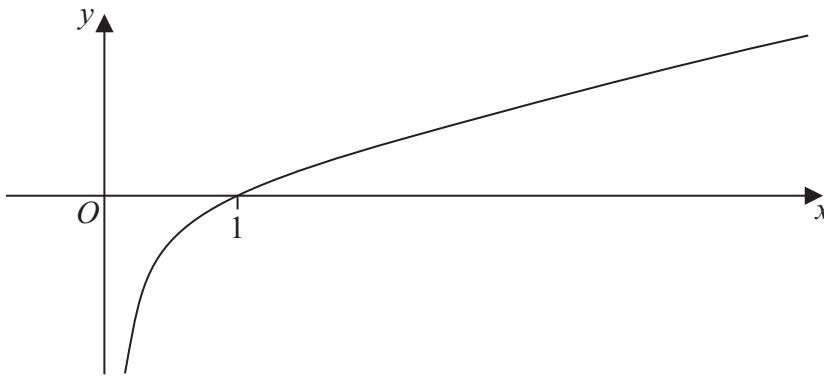


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x . The curve crosses the x -axis at the point $(1, 0)$ and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$ **(2)**

(b) $y = |f(x)|$, $x > 0$ **(3)**

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x -axis.



3.

$$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the exact value of R and the value of α to one decimal place.

(3)

(b) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leq x < 360^\circ$, giving your answers to one decimal place.

(5)

(c) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leq x < 360^\circ$

(2)



Leave
blank

5. (a) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to x .

(3)

(b) Show that $\frac{d}{dx}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant.

(3)

(c) Given $x = 2 \sin\left(\frac{y}{3}\right)$, find $\frac{dy}{dx}$ in terms of x , simplifying your answer.

(4)



6. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ .

(3)

- (ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π .

(6)



7.

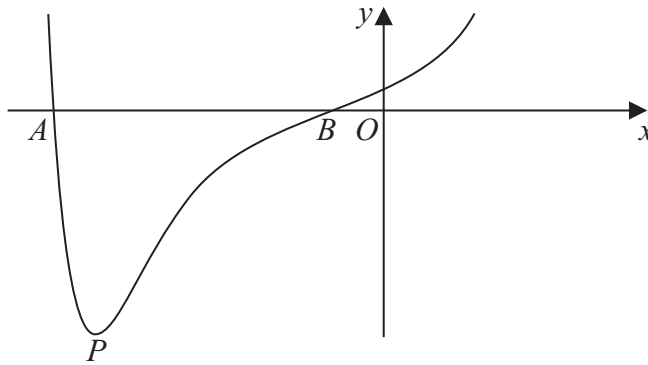


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x -axis at points A and B as shown in Figure 2.

- (a) Calculate the x coordinate of A and the x coordinate of B , giving your answers to 3 decimal places. (2)

- (b) Find $f'(x)$. (3)

The curve has a minimum turning point at the point P as shown in Figure 2.

- (c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)} \tag{3}$$

- (d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

The x coordinate of P is α .

- (e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places. (2)



8.

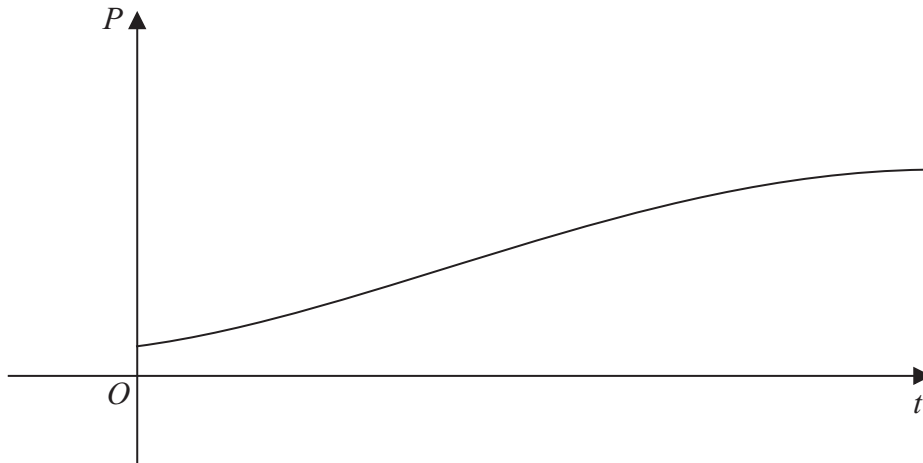


Figure 3

The population of a town is being studied. The population P , at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \quad t \geq 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study, (2)

(b) find a value for the expected upper limit of the population. (1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places. (5)

Using this value for k ,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures. (2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study. (3)



Leave
blank

Question 8 continued

Lined area for writing the answer to Question 8.

(Total 13 marks)

Q8

--	--

TOTAL FOR PAPER: 75 MARKS

END



2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

(7)



3. Given that

$$2 \cos(x + 50)^\circ = \sin(x + 40)^\circ$$

(a) Show, without using a calculator, that

$$\tan x^\circ = \frac{1}{3} \tan 40^\circ \quad \text{(4)}$$

(b) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos(2\theta + 50)^\circ = \sin(2\theta + 40)^\circ$$

giving your answers to 1 decimal place. (4)



4. $f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$. **(5)**

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5} e^{-x}$. **(1)**

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5} e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. **(3)**

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer. **(2)**



5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x . Give your answer in its simplest form.

(4)



7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

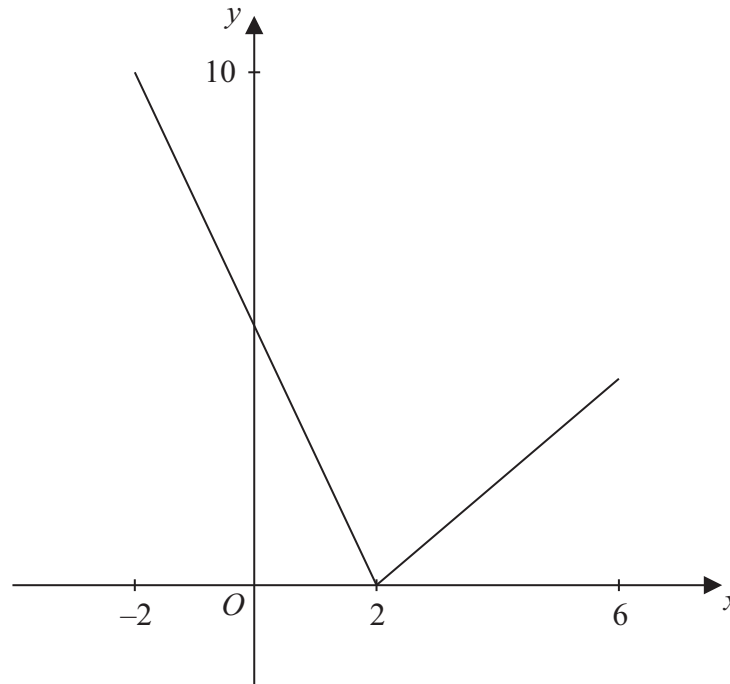


Figure 1

- (a) Write down the range of f . (1)
- (b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$ (3)
- (d) Solve the equation $gf(x) = 16$ (5)



8.

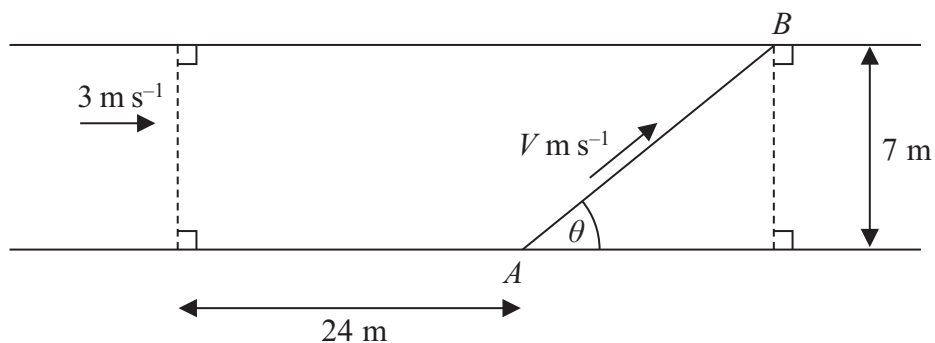


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.

John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

- (b) find the minimum value of V . (2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB . (3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$. (6)



Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$